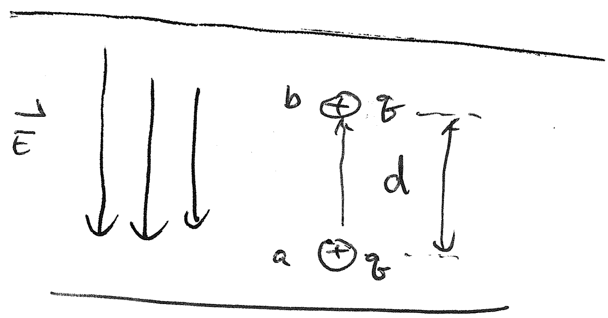
WEEK 3

WED

ELECTRICAL POTENTIAL ENERGY

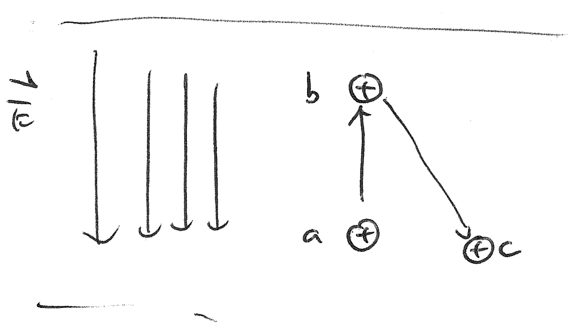


~~WORK DONE BY ELECTRIC FIELD~~

$\cancel{W_{a \to b} = -qEd}$

WORK DONE BY MY HAND

$W = qEd$



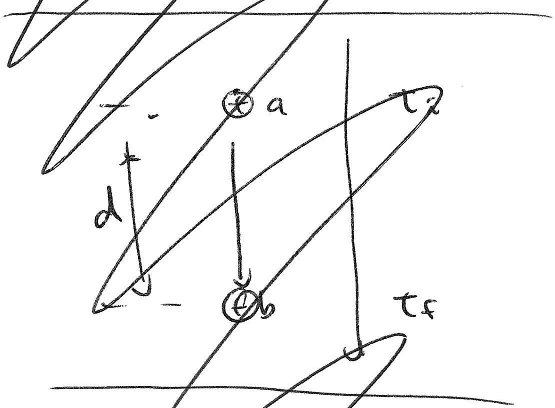
WORK DONE

$W = qEd - qEd = 0$

WORK ONLY DEPENDS ONLY ON FINAL POSITONS

ELECTRIC FIELD EXERTS CONSERVATIVE FORCES

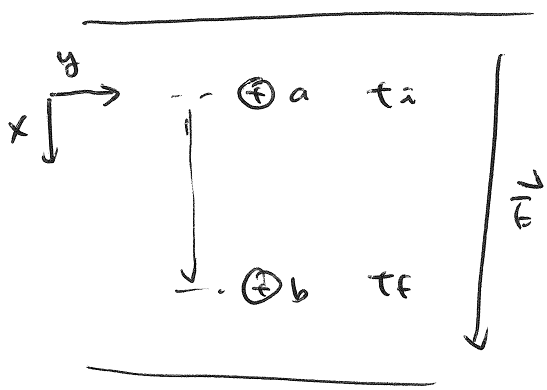
EXAMPLE



$V(t_i) = 0$

$V(t_f) = ?$

NOW LET'S REVERSE THINKING A LITTLE



$V(t_i) = 0$ AT REST

$\vec{F} = q\vec{E}$

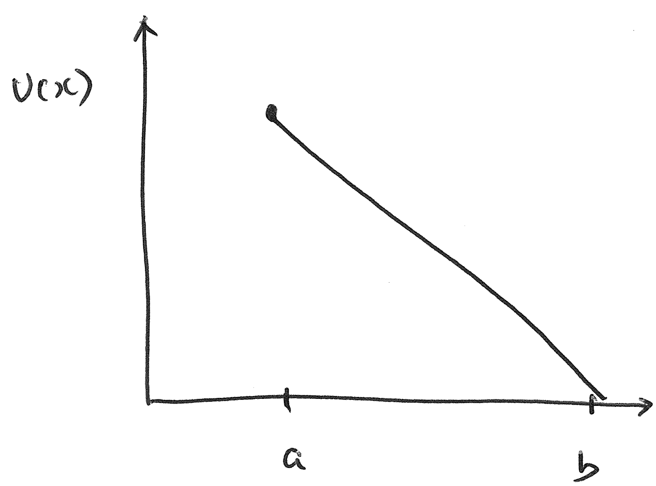
$W_{a\to b}$ = WORK DONE BY THE FIELD BETWEE $a \to b$

$= qEd$

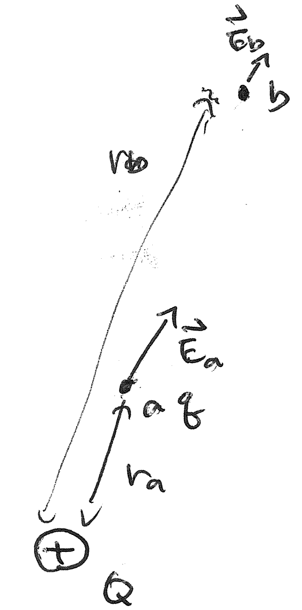
IN CONSERVATIVE FIELD

$W_{a\to b} = qEd = \underbrace{U_a - U_b}_{=}$

POTENTIAL ENERGY



ELECTRIC POTENTIAL ENERGY



WORK DONE BY THE FIELD

$$\int_{r_a}^{r_b} \vec{F}\cdot d\vec{r}$$

$$= \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\,dr$$

$$= \frac{Qq}{4\pi\varepsilon_0} \; -\frac{1}{r}\Big]_{r_a}^{r_b}$$

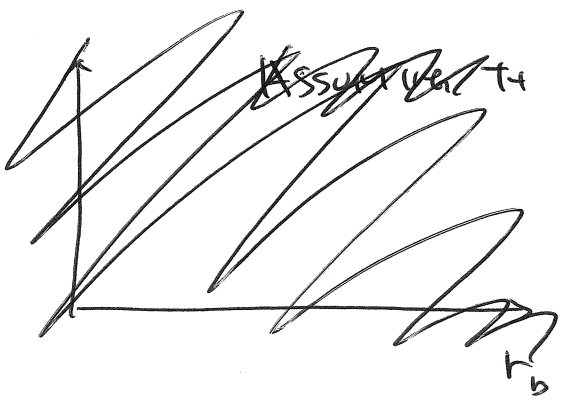
$$= \frac{Qq}{4\pi\varepsilon_0}\left[-\frac{1}{r_b} - \left(-\frac{1}{r_a}\right)\right]$$

$$W_{a\to b} = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

BY THE FIELD

$$W_{a\to b} = \frac{Qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] = U_a - U_b$$

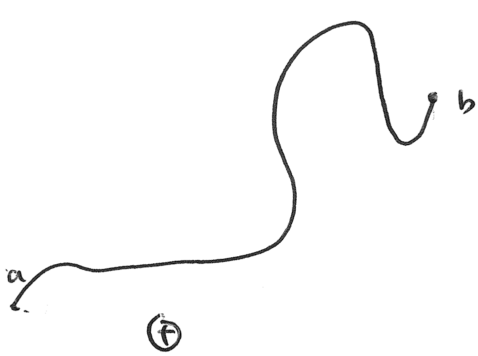
$$\therefore \cancel{U(r) = \frac{Qq_0}{4\pi\varepsilon_0}\frac{q q_0}{r}}$$



TAKE LIMIT $r_b \to \infty$

$U(r_b) = 0$ 0 POTENTIAL ENERGY REALLY REALLY FAR AWAY

$$\boxed{U(r) = \frac{Q}{4\pi\varepsilon_0}\frac{q_0}{r}}$$ [JOULES]



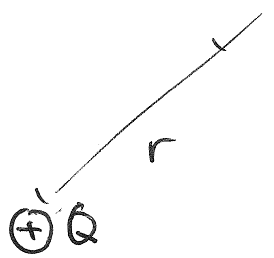
$$W_{a\to b} = \frac{Qq_0}{4\pi}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

ELECTRIC POTENTIAL

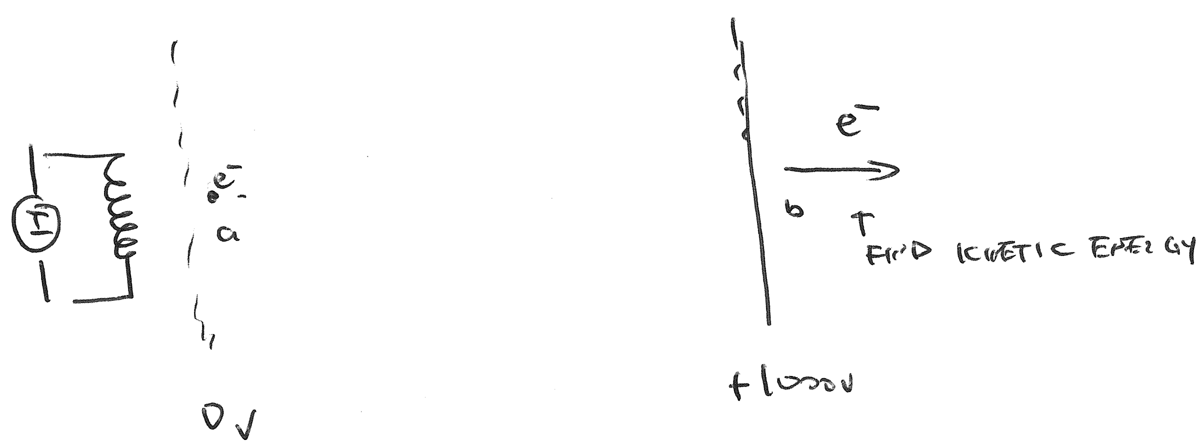
$$V = \frac{U}{q} \qquad \underline{U = qV}$$

POTENTIAL ENERGY PER UNIT CHARGE

V : UNITS <u>VOLT</u>



$$V(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r} \quad [\text{VOLT}]$$



$$U(a) - U(b) = q\left[V(a) - V(b)\right] = W_{a\to b} = K.E(b)$$

$$\cancel{\frac{V(a) - V(b)}{-1.6\times10^{-19}\,C} = \frac{-1000V}{-1.6\times10^{-19}\,C} = K.E \quad K.E = \frac{1000V}{1.6}}$$

$$q\,[V(a) - V(b)] = -1.6 \times 10^{-19}\,C \cdot -1000\,V$$

$$= 1.6 \times 10^{-1\cancel{9}6} \cdot V \cdot C$$

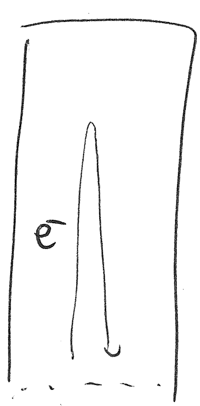
$$= 1.6 \times 10^{-16}\,J$$

OR

$1000\,eV$

$1\,eV = 1.6 \times 10^{-19}\,J$

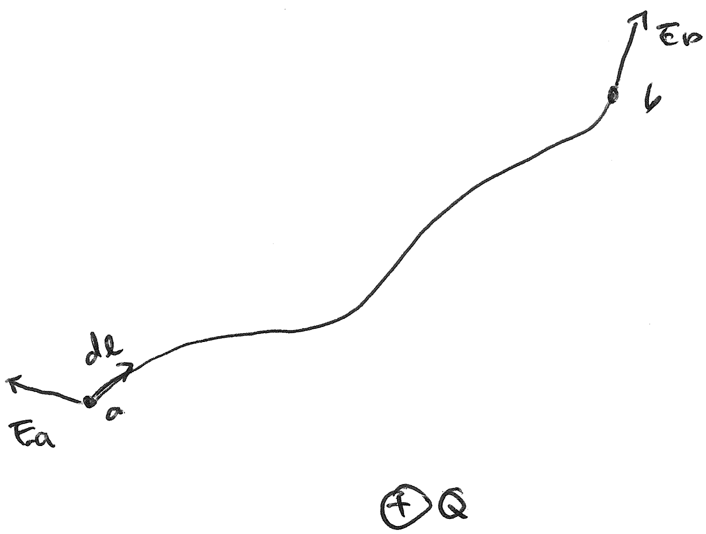
ASIDE



MASS OF ELECTRON IN ACCELERATED FRAME VS. INERTIAL FRAM

WED / FRI

REVAMP OUR UNDERSTANDING



q : "TEST" CHARGE

$W_{a \to b}$

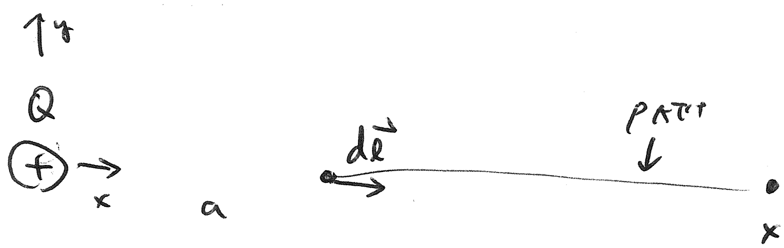
(WORK DONE BY ~~E~~ ELECTRIC FIELD ON CHARGE q)

$$= \int_{a \atop \text{PATH}}^{b} q\vec{E} \cdot d\vec{\ell} = U(a) - U(b) = \boxed{V(a) - V(b)}\, q$$

$$V(a) - V(b) = \int_{a\,(\text{PATH})}^{b} \vec{E} \cdot d\vec{\ell}$$

$$V(b) - V(a) = -\int_{a\,(\text{PATH})}^{b} \vec{E} \cdot d\vec{\ell}$$

SOUNDS HARD BUT CONSIDER



$$d\vec{l} = dr\,\hat{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$V(x) - V(a) = -\int_a^x \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot dr\,\hat{r}$$

$$= -\frac{1}{4\pi\varepsilon_0}\,Q\int_a^x \frac{dr}{r^2}\,\cancelto{1}{\hat{r}\cdot\hat{r}}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left.-\frac{1}{r}\right]_a^x$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x} - \frac{1}{a}\right] < 0$$

OK.

$$\frac{dV}{dx} = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{x^2} = -E_x(x)$$

$$\vec{E} = -\vec{\nabla}V = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

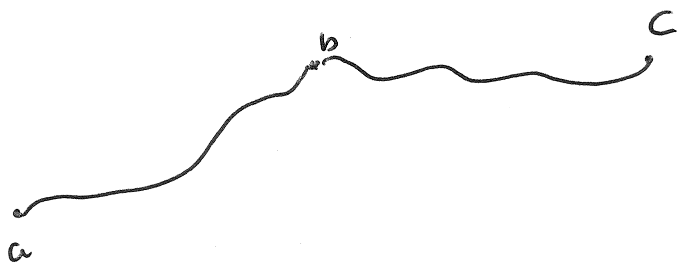
NOTE

1. V : POTENTIAL NOT POTENTIAL ENERGY

2. $\vec{E} = -\vec{\nabla} V$

$$\vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \vec{\nabla} V = 0$$

3.



$$V(c) - V(a) = -\int_a^c \vec{E} \cdot d\vec{l}$$

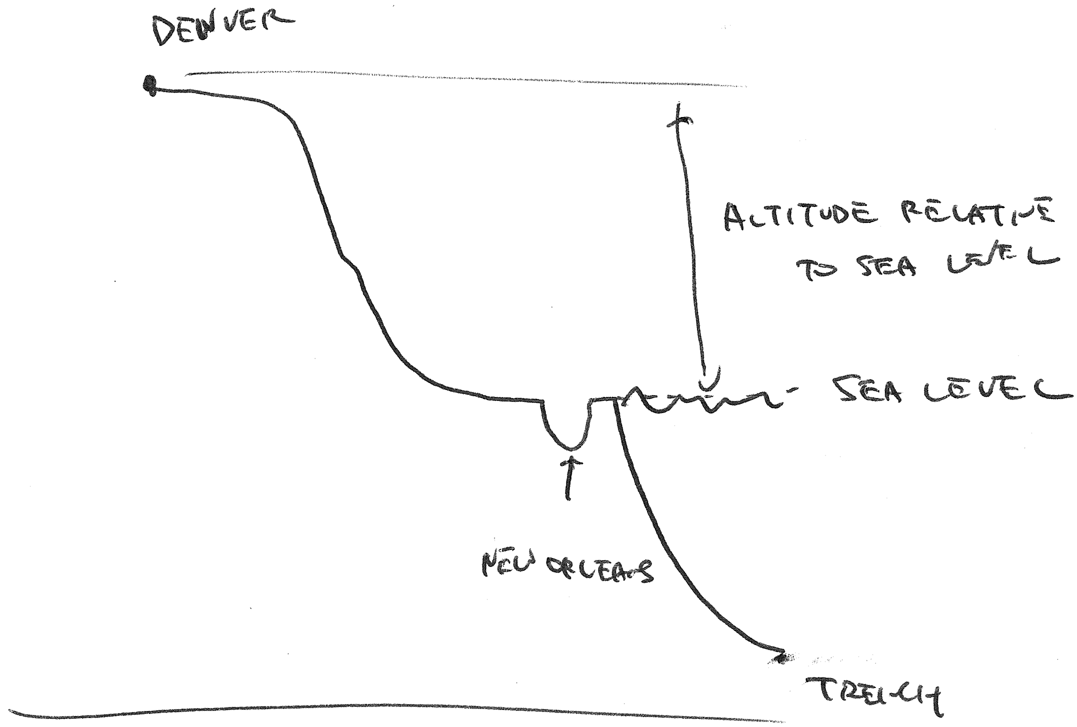
$$= -\int_a^b \vec{E} \cdot d\vec{l} - \int_b^c \vec{E} \cdot d\vec{l}$$

$$V(c) - V(a) = V(b) - V(a) - \left[V(c) - V(b) \right]$$

i.e.

$(V_{ca} + \text{CONSTANT} = V_{bc})$ IT'S ALL RELATIVE TO WHERE YOU START

MUCH LIKE ALTITUDE



TO STANDARIZE WE SET $V=0$ AT ∞

$V(\infty)=0$

$\oint_1$ LAW OF SUPERPOSITION STILL APPLIES

q TEST CHARGE

q_1

q_2

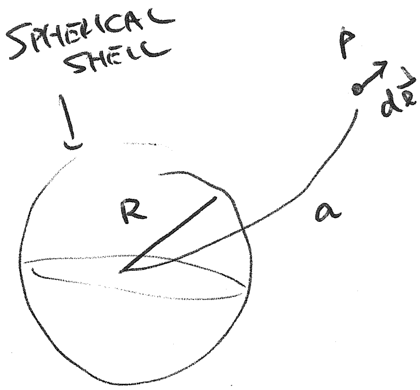
q_3

$F_T = F_1 + F_2 + F_3$

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$V = V_1 + V_2 + V_3$

EXAMPLE



OUTSIDE

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

INSIDE : NO CHARGE IS ENCLOSED

$$\vec{E} = 0$$

POTENTIAL :

$$V(\infty) - V(a) = 0 - V(a) = -\int_a^\infty \vec{E}\cdot d\vec{l}$$

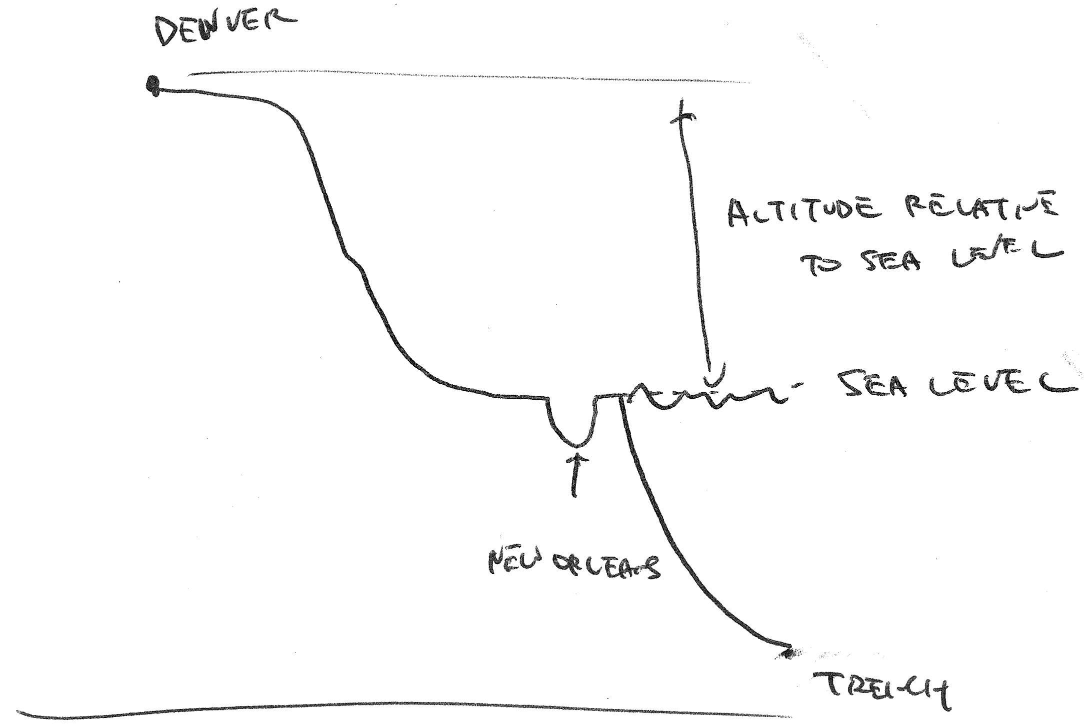
$$= -\int_a^\infty \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr\,\overset{1}{\cancel{\hat{r}\cdot\hat{r}}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\Bigg]_a^\infty$$

$$-V(a) = \frac{1}{4\pi\varepsilon_0}\,q\,\left[\frac{1}{\infty} - \frac{1}{a}\right)$$

$$V(a) = \frac{q}{4\pi\varepsilon_0}\frac{1}{a} \quad (\text{FOR } a > R)$$

MUCH LIKE ALTITUDE



TO STANDARIZE WE SET $V=0$ AT ∞

$V(\infty)=0$

4. LAW OF SUPERPOSITION STILL APPLIES

q TEST CHARGE

q_1

q_2

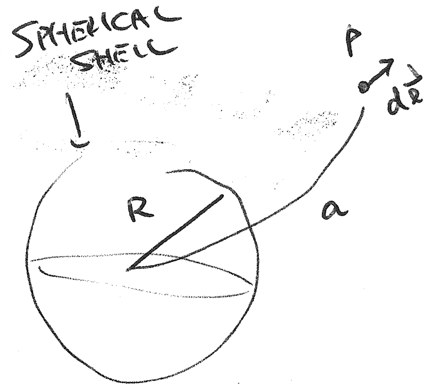
q_3

$F_T = F_1 + F_2 + F_3$

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$V = V_1 + V_2 + V_3$

EXAMPLE



OUTSIDE

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

INSIDE : NO CHARGE IS ENCLOSED

$$\vec{E} = 0$$

POTENTIAL :

$$V(\infty) - V(a) = 0 - V(a) = -\int_a^\infty \vec{E}\cdot d\vec{\ell}$$

$$= -\int_a^\infty \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr\,\overset{1}{\cancel{\hat{r}\cdot\hat{r}}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\Big]_a^\infty$$

$$-V(a) = \frac{1}{4\pi\varepsilon_0}\,q\left[\frac{1}{\infty} - \frac{1}{a}\right)$$

$$V(a) = \frac{q}{4\pi\varepsilon_0}\frac{1}{a} \quad (\text{FOR } a > R)$$

INSIDE : $a < R$

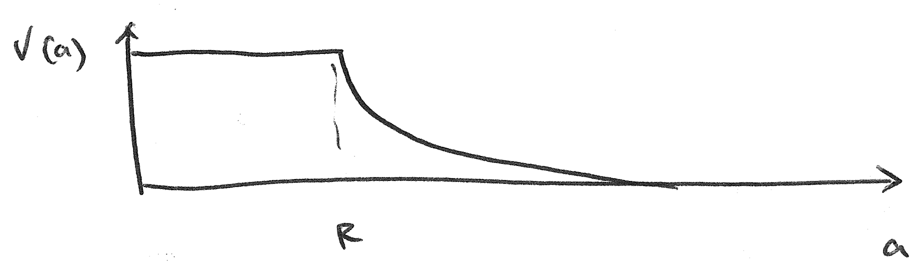
$$V(\infty) - V(a) = V(\infty) - V(R) + V(R) - V(a)$$

$$= 0 - \frac{1}{4\pi\varepsilon_0}\frac{q}{R} - \int_a^R \overset{0}{\cancel{\vec{E}}} \cdot d\vec{l}$$

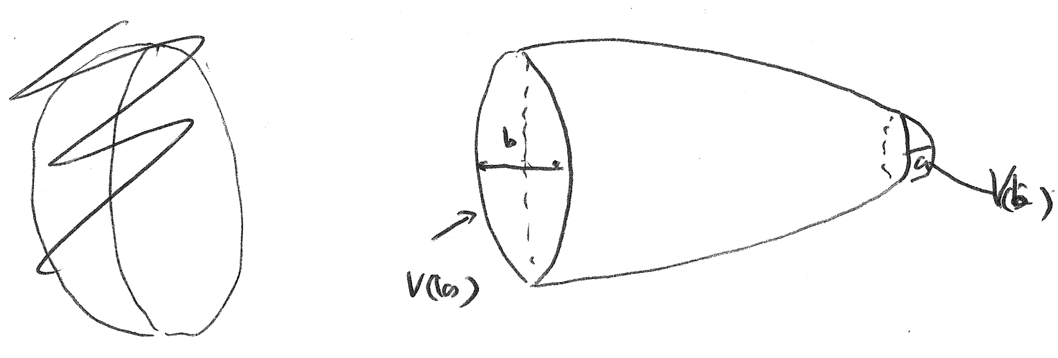
$E = 0$ INSIDE

$$-V(a) = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$V(a) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \quad (\text{FOR } a < R)$$



ODD SHAPED OBJECT (METALLIC SHELL



IF $V(a) \neq V(b)$ CHARGE CAN MOVE AROUND

SO AT EQUILIBRIUM

$$V(a) = V(b)$$

$$\frac{q_a}{a} = \frac{q_b}{b}$$

$$\frac{q_a}{q_b} = \frac{a}{b}$$

$$\sigma_a \cong \frac{q_a}{a^2} \qquad \sigma_b \cong \frac{q_b}{b^2}$$

$$\frac{a^2 \sigma_a}{b^2 \sigma_b} \approx \frac{a}{b}$$

$$\frac{\sigma_a}{\sigma_b} = \frac{b}{a}$$

$$\underline{\sigma_a \gg \sigma_b}$$

$$b \gg a$$

CHARGE IS CONCENTRATED AT THE POINT OF LARGE ~~CURVATR~~ CURVATURE)

CORONA DISCHARGE : AIR CAN SUPPORT ONLY UP TO

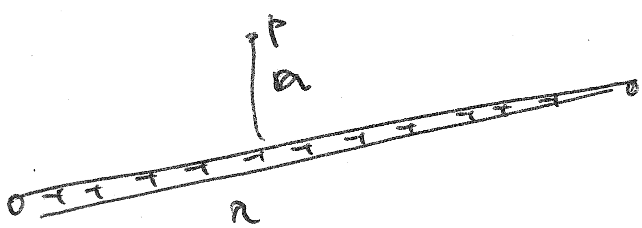
3×10^6 V/m

$$E_{surface} = \frac{V_{surface}}{R}$$

$$3\times10^6 \text{ V/m} \cdot 0.01 = \underline{3\times10^4 \text{ V}}$$

DEMO REQUEST DISCHARGE

EXAMPLE #2



$$E(a) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a}\hat{r} \quad \text{(Gauss's law} \qquad d\vec{l} = dr\,\hat{r}$$

$$V(\infty) - V(a) = -\int_{a}^{\infty} \vec{E}\cdot d\vec{l}$$

$$= -\int_{a}^{\infty} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}\cdot dr\,\hat{r}$$

$$= -\int_{a}^{\infty} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\,dr$$

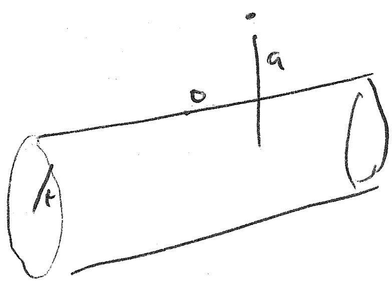
$$0 - V(a) = - \int_a^{\infty} \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r}\, dr$$

$$= - \frac{\lambda}{2\pi\varepsilon_0} \ln r \Big]_a^{\infty}$$

$$V(a) = \frac{\lambda}{2\pi\varepsilon_0} \left[\infty - \ln a \right]$$

A PROBLEM

BUT WE CAN SET $V=0$ AT ANY POINT



$$V(a) - V(R) = - \int_R^a \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r}\, dr$$

$$V(a) - 0 = - \frac{\lambda}{2\pi\varepsilon_0} \ln r \Big]_R^a$$

$$= - \frac{\lambda}{2\pi\varepsilon_0} \left(\ln a - \ln R \right)$$

$$V(a) = - \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{a}{R} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{R}{a}\right)$$